Mathematics — Solved Paper 2018

SECTION A (40 Marks)

*(Answer **all** questions from this Section)*

***Question* 1 :**

(a) Find the values of 'x' and 'y' if :

$$2\begin{bmatrix} x & 7 \\ 9 & y-5 \end{bmatrix} + \begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}.$$ **[3]**

Solution :

$$2\begin{bmatrix} x & 7 \\ 9 & y-5 \end{bmatrix} + \begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 2x & 14 \\ 18 & 2y-10 \end{bmatrix} + \begin{bmatrix} 6 & -7 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 2x+6 & 7 \\ 22 & 2y-5 \end{bmatrix} = \begin{bmatrix} 10 & 7 \\ 22 & 15 \end{bmatrix}$$

$\Rightarrow \quad 2x + 6 = 10$ and $2y - 5 = 15$

$\Rightarrow \quad \boldsymbol{x = 2}$ and $\boldsymbol{y = 10}$ **Ans.**

(b) Sonia had a recurring deposit account in a bank and deposited ₹ 600 per month for $2\frac{1}{2}$ years. If the rate of interest was 10% p.a., find the maturity value of this account. [3]

Solution :

Given monthly instalment, P = ₹ 600

Number of months, $n = 2\frac{1}{2}$ years $\times 12 = 30$

and rate of interest, $r = 10\%$ p.a.

$\therefore$ **Maturity value** $= P \times n + P \times \dfrac{n(n+1)}{2\times 12} \times \dfrac{r}{100}$

$= ₹\ 600 \times 30 + ₹\ 600 \times \dfrac{30\times 31}{2\times 12} \times \dfrac{10}{100}$

= ₹ 18,000 + ₹ 2,325 = **₹ 20,325** **Ans.**

(c) Cards bearing numbers 2, 4, 6, 8, 10, 12 14, 16, 18 and 20 are kept in a bag. A card is drawn at random from the bag. Find the probability of getting a card which is :

(i) a prime number. **(ii) a number divisible by 4.**

(iii) a number that is a multiple of 6. **(iv) an odd number. [4]**

Solution :

Clearly, total number of outcomes = 10 [2, 4, 6, 8, 10, 12, 14, 16, 18 and 20]

(i) **Favourable outcomes**

= Number of prime number

= 1 [2 is a prime number]

$\therefore$ **Probability of getting a prime number**

$= \mathbf{\frac{1}{10}}$ **Ans.**

(ii) **Favourable outcomes**

= Number of numbers divisible by 4

= 5 [4, 8, 12, 16 and 20]

$\therefore$ **Required probability**

$= \frac{5}{10} = \mathbf{\frac{1}{2}}$ **Ans.**

(iii) **Favourable outcomes**

= Number of numbers multiples of 6

= 3 [6, 12 and 18]

$\therefore$ **Required probability**

$= \mathbf{\frac{3}{10}}$ **Ans.**

(iv) **Favourable outcomes**

= Number of odd numbers

= 0 [all the given numbers are even]

$\therefore$ **Required probability**

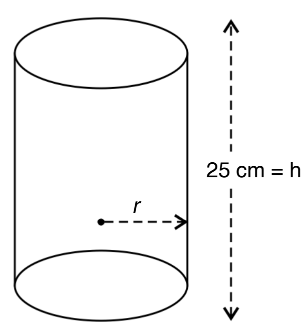
$= \frac{0}{10} = \mathbf{0}$ **Ans.**

***Question* 2 :**

(a) The circumference of the base of a cylinderical vessel is 132 cm and its height is 25 cm. Find the

(i) radius of the cylinder.

(ii) volume of the cylinder. (use $\pi = \frac{22}{7}$)



Solution :

(i) Since, for the given cylindrical vessel,

curcumference = 132 cm

$\Rightarrow \quad 2\pi r = 132$

$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 132$

$\Rightarrow$ **Radius**, $r = \frac{132 \times 7}{2 \times 22}$ cm

= **21 cm** **Ans.**

(ii) **Volume of cylinder** $= \pi r^2 h$

$= \frac{22}{7} \times 21 \times 21 \times 25 \text{ cm}^3$ [Given, h = 25 cm]

$= \mathbf{34{,}650\ cm^3}$ **Ans.**

(b) If (k – 3), (2k + 1) and (4k + 3) are three consecutive terms of an A.P., find the value of k. **[3]**

Solution :

(k – 3), (2k + 1) and (4k + 3) are consecutive terms of an A.P.

$\Rightarrow \quad (2k + 1) - (k - 3) = (4k + 3) - (2k + 1)$

$\Rightarrow \quad 2k + 1 - k + 3 = 4k + 3 - 2k - 1$

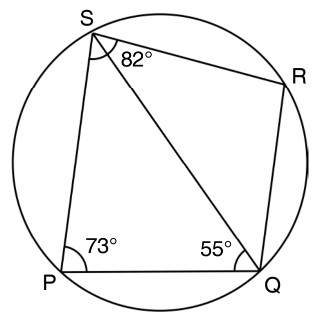
$\Rightarrow \quad k + 4 = 2k + 2$ *i.e.* $\mathbf{k = 2}$ **Ans.**

(c) PQRS is a cyclic quadrilateral. Given ∠QPS = 73°, ∠PQS = 55° and ∠PSR = 82°, calculate :

(i) ∠QRS

(ii) ∠RQS

(iii) ∠PRQ



Solution :

(i) Since, opposite angles of a cyclic quadrilateral are supplementry

$\therefore \quad \angle P + \angle QRS = 180°$

$\Rightarrow \quad 73° + \angle QRS = 180°$

i.e. $\quad \angle \mathbf{QRS} = 180° - 73° = \mathbf{107°}$ **Ans.**

(ii) For the same reason :

$\angle PSR + \angle PQR = 180°$

$\Rightarrow 82° + (55° + \angle RQS) = 180°$

$\Rightarrow \quad \angle \mathbf{RQS} = 180° - 82° - 55° = \mathbf{43°}$ **Ans.**

(iii) Since, angles in the same-segment are equal

$\therefore \quad \angle \mathbf{PRQ} = \angle PSQ$

$= 180° - (73° + 55°) = \mathbf{52°}$ **Ans.**

***Question* 3 :**

(a) If (x + 2) and (x + 3) are factors of $x^3 + ax + b$, find the values of 'a' and 'b'. **[3]**

Solution :

Since, (x + 2) is a factor of $x^3 + ax + b$

$\Rightarrow \quad (-2)^3 + a(-2) + b = 0$ $\quad [x + 2 = 0 \Rightarrow x = -2]$

$\Rightarrow \quad -8 - 2a + b = 0$ *i.e.* $-2a + b = 8$ I

Since, (x + 3) is a factor of $x^3 + ax + b$

$\Rightarrow \quad (-3)^3 + a(-3) + b = 0$ $\quad [x + 3 = 0 \Rightarrow x = -3]$

$\Rightarrow \quad -27 - 3a + b = 0$ *i.e.* $-3a + b = 27$ II

On solving equations I and II, we get :

$\mathbf{a = -19}$ and $\mathbf{b = -30}$ **Ans.**

(b) Prove that $\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta}$ = tan θ + cot θ. **[3]**

Solution :

$$\text{L.H.S.} = \sqrt{\sec^2\theta + \operatorname{cosec}^2\theta}$$

$$= \sqrt{1+\tan^2\theta + 1 + \cot^2\theta}$$

$$= \sqrt{\tan^2\theta + \cot^2\theta + 2}$$

$$= \sqrt{\tan^2\theta + \cot^2\theta + 2\tan\theta\cdot\cot\theta} \qquad \left[\tan\theta\cdot\cot\theta = \tan\theta\times\frac{1}{\tan\theta} = 1\right]$$

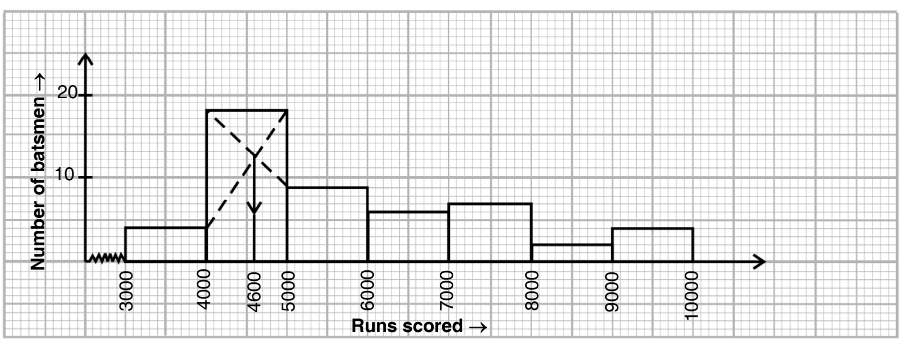
$$= \sqrt{(\tan\theta+\cot\theta)^2} = \tan\theta + \cot\theta = \textbf{R.H.S.}$$ **Ans.**

(c) Using a graph paper draw a histogram for the given distribution showing the number of runs scored by 50 batsmen. Estimae the mode of the data : **[4]**

Runs scored	3000-4000	4000-5000	5000-6000	6000-7000	7000-8000	8000-9000	9000-10000
No. of batsmen	4	18	9	6	7	2	4

Solution :

Using the given data, draw a histogram as shown below :



Clearly, **the mode of the given data = 4600** **Ans.**

***Question* 4 :**

(a) Solve the following inequation, write down the solution set and represent it on the real number line :

$-2 + 10x \le 13x + 10 < 24 + 10x,\ x \in Z$

Solution :

$-2 + 10x \le 13x + 10 < 24 + 10x,\ x \in Z$

$\Rightarrow \quad -2 + 10x \le 13x + 10 \quad$ and $\quad 13x + 10 < 24 + 10x$

$\Rightarrow \quad -2 - 10 \le 13x - 10x \quad$ and $\quad 13x - 10x < 24 - 10$

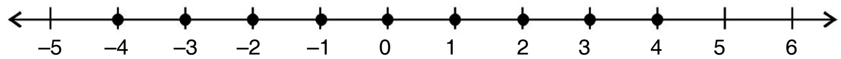
$\Rightarrow \quad -12 \le 3x \quad$ and $\quad 3x < 14$

$\Rightarrow \quad -4 \le x \quad$ and $\quad x < \frac{14}{3}$

i.e. $\quad -4 \le x < 4\frac{2}{3}$

$\because \quad x \in Z$, the **solution set = {–4, –3, –2, –1, 0, 1, 2, 3, 4}** **Ans.**

Solution on the real number line.

= 

Ans.

(b) If the straight lines 3*x* – 5*y* = 7 and 4*x* + *ay* + 9 = 0 are perpendicular to one another, find the value of *a*. **[3]**

Solution :

$3x - 5y = 7 \quad \Rightarrow \quad -5y = -3x + 7$

$\Rightarrow \quad y = \frac{3x}{5} - \frac{7}{5}$

$\Rightarrow$ Its slope, $m_1 = \frac{3}{5}$

$4x + ay + 9 = 0 \quad \Rightarrow \quad ay = -4x - 9$

$\Rightarrow \quad y = \frac{-4}{a}x - \frac{9}{a}$

$\Rightarrow$ Its slope, $m_2 = -\frac{4}{a}$

$\because$ Given lines are perpendicular to each other

$\Rightarrow m_1 \times m_2 = -1 \quad$ *i.e.* $\quad \frac{3}{5} \times -\frac{4}{a} = -1$

$\Rightarrow \quad \frac{-12}{5a} = -1$

$\Rightarrow \quad -5a = -12$ *i.e.* $\boldsymbol{a} = \frac{12}{5} = \mathbf{2\frac{2}{5}}$ **Ans.**

(c) Solve $x^2 + 7x = 7$ and give your answer correct to two decimal places. **[4]**

Solution :

$\mathbf{x^2 + 7x = 7} \quad \Rightarrow \quad x^2 + 7x - 7 = 0$

Comparing $x^2 + 7x - 7 = 0$, with $ax^2 + bx + c = 0$, we get :

$a = 1$, $b = 7$ and $c = -7$

$$\therefore \quad \boldsymbol{x} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-7 \pm \sqrt{49 - 4 \times 1 \times -7}}{2 \times 1}$$

$$= \frac{-7 \pm \sqrt{77}}{2}$$

$$= \frac{-7 \pm 8{\cdot}775}{2}$$

$= \dfrac{-7 + 8{\cdot}775}{2}$ or $\dfrac{-7 - 8{\cdot}775}{2}$

$= \dfrac{1{\cdot}775}{2}$ or $\dfrac{-15{\cdot}775}{2}$

$= 0{\cdot}8875$ or $-7{\cdot}8875$

= **0·89** or **–7·89** (correct to 2 decimal places) **Ans.**

SECTION B (40 Marks)

(Answer any ***four*** *questions from this Section)*

Question **5 :**

(a) The 4th term of a G.P. is 16 and the 7th term is 128. Find the first term and common ratio of the series. **[3]**

Solution :

Let the first term = a and the common ratio = r

4th term = 16 $\Rightarrow$ $ar^3 = 16$ and **7th term = 128** $\Rightarrow$ $ar^6 = 128$

So, $\dfrac{ar^6}{ar^3} = \dfrac{128}{16} \Rightarrow r^3 = 8$ and $r = 2$

Now, $ar^3 = 16 \Rightarrow a \times 2^3 = 16$ *i.e.* $a = \dfrac{16}{8} = 2$

$\therefore$ The **first term = 2** and **the common ratio = 2** **Ans.**

(b) A man invests ₹ 22,500 in ₹ 50 shares available at 10% discount. If the dividend paid by the company is 12%, calculate :

(i) The number of shares purchased.

(ii) The annual dividend received.

(iii) The rate of return he gets on his investment. Give your answer correct to the nearest whole number. **[3]**

Solution :

Since, sum invested = ₹ 22,500, N.V. of each share = ₹ 50

and M.V. of each share = ₹ 50 – 10% of ₹ 50 = ₹ 45

(i) **The number of shares purchased**

$= \dfrac{\text{Sum invested}}{\text{M.V. of each share}} = \dfrac{₹\ 22{,}500}{₹\ 45} = \mathbf{500}$ **Ans.**

(i) **The annual dividend**

= Dividend on 1 share × number of shares

= (12% of ₹50) × 500 = **₹ 3,000** **Ans.**

(iii) If rate of return = x%

x% of ₹ 45 = 12% of ₹ 50

$\Rightarrow \dfrac{x \times 45}{100} = \dfrac{12 \times 50}{100}$ *i.e.* $x = \dfrac{600}{45} = 13{\cdot}33$

$\therefore$ **Required rate of return correct to the nearest whole number = 13%** **Ans.**

(c) Use graph paper for this question (Take 2 cm = 1 unit along both x and y axes). ABCD is a quadrilateral whose vertices are A(2, 2), B(2, –2), C(0, –1) and D(0, 1).

(i) Reflect quadrilateral ABCD on the y-axis and name it as A′B′CD.

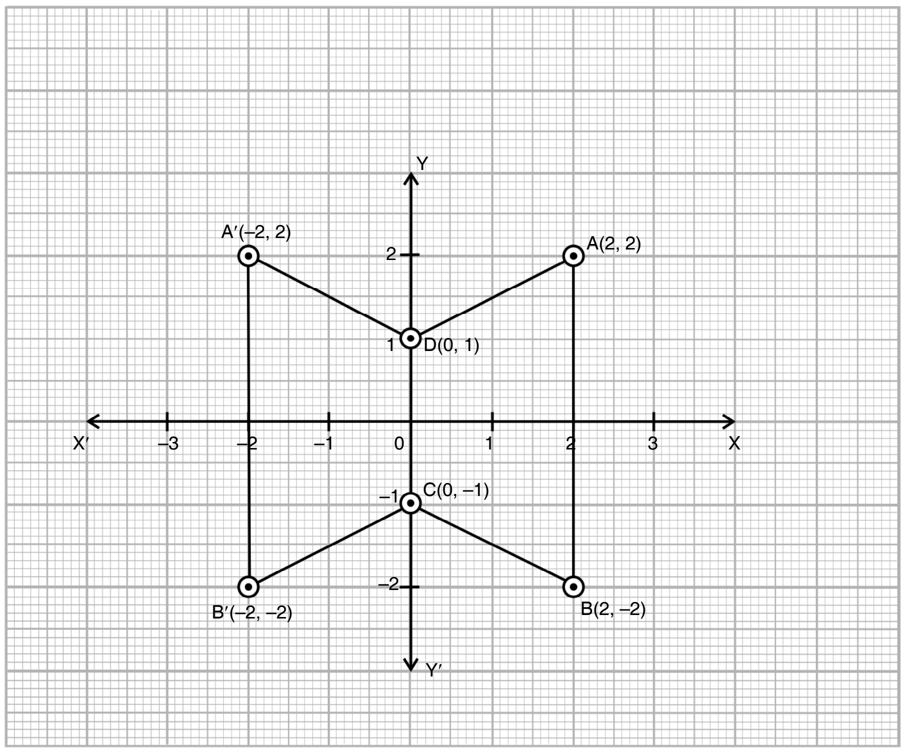
(ii) Write down the coordinates of A′ and B′.

(iii) Name two points which are invariant under the above reflection.

(iv) Name the polygon A′B′CD. **[4]**

Solution :

(i)



(ii) **A′ = (–2, 2)** and **B′ = (–2, –2)** **Ans.**

(iii) **C(0, –1)** and **D(0, 1)** **Ans.**

(iv) **Trapezium.** **Ans.**

***Question* 6 :**

(a) Using properties of proportion, solve for x. Given that x is positive : **[3]**

$$\frac{2x+\sqrt{4x^2-1}}{2x-\sqrt{4x^2-1}} = 4.$$

Solution :

Given $\frac{2x+\sqrt{4x^2-1}}{2x-\sqrt{4x^2-1}} = \frac{4}{1}$ $\left[4=\frac{4}{1}\right]$

$$\therefore \quad \frac{2x+\sqrt{4x^2-1}+2x-\sqrt{4x^2-1}}{2x+\sqrt{4x^2-1}-2x+\sqrt{4x^2-1}} = \frac{4+1}{4-1}$$ [Applying componendo and dividendo]

$$\Rightarrow \quad \frac{4x}{2\sqrt{4x^2-1}} = \frac{5}{3}$$

$$\Rightarrow \quad \frac{2x}{\sqrt{4x^2-1}} = \frac{5}{3}$$

Squaring both the sides, we get :

$$\frac{4x^2}{4x^2-1} = \frac{25}{9} \quad \textit{i.e.} \quad 36x^2 = 100x^2 - 25$$

$$\Rightarrow \quad 64x^2 = 25 \quad \textit{i.e.} \quad x^2 = \frac{25}{64}$$

$\Rightarrow$ $\boldsymbol{x = \frac{5}{8}}$, *because x is positive.* **Ans.**

(b) If A = $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$, B = $\begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$ and C = $\begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix}$, find AC + B^2 – 10C. **[3]**

Solution :

$$AC = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 2-3 & 0+12 \\ 5-7 & 0+28 \end{bmatrix} = \begin{bmatrix} -1 & 12 \\ -2 & 28 \end{bmatrix}$$

$$B^2 = B \times B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} = \begin{bmatrix} 0-4 & 0+28 \\ 0-7 & -4+49 \end{bmatrix} = \begin{bmatrix} -4 & 28 \\ -7 & 45 \end{bmatrix}$$

$$10C = 10\begin{bmatrix} 1 & 0 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ -10 & 40 \end{bmatrix}$$

$$\therefore \quad \mathbf{AC + B^2 - 10C} = \begin{bmatrix} -1 & 12 \\ -2 & 28 \end{bmatrix} + \begin{bmatrix} -4 & 28 \\ -7 & 45 \end{bmatrix} - \begin{bmatrix} 10 & 0 \\ -10 & 40 \end{bmatrix}$$

$$= \begin{bmatrix} -1-4-10 & 12+28-0 \\ -2-7+10 & 28+45-40 \end{bmatrix} = \begin{bmatrix} \mathbf{-15} & \mathbf{40} \\ \mathbf{1} & \mathbf{33} \end{bmatrix}$$ **Ans.**

(c) Prove that (1 + cot θ – cosec θ) (1 + tan θ + sec θ) = 2. **[4]**

Solution :

L.H.S. = (1 + cot θ – cosec θ) (1 + tan θ + sec θ)

$$= \left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)\left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)$$

$$= \left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)\left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)$$

$$= \frac{(\sin\theta+\cos\theta)^2-(1)^2}{\sin\theta\cos\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta}$$

$$= \frac{1 + 2\sin\theta\cos\theta - 1}{\sin\theta\cos\theta} \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = 2 = \textbf{R.H.S.}$$ **Hence Proved**

***Question* 7 :**

(a) Find the value of *k* for which the following equation has equal roots. [3]

$$\boldsymbol{x^2 + 4kx + (k^2 - k + 2) = 0}$$

Solution :

Comparing given equation $x^2 + 4kx + (k^2 - k + 2) = 0$ with equation $ax^2 + bx + c = 0$; we get :

$a = 1$, $b = 4k$ and $c = k^2 - k + 2$.

$\because$ Roots are equal

$\Rightarrow \quad b^2 - 4ac = 0$

$\Rightarrow \quad (4k)^2 - 4 \times 1 \times (k^2 - k + 2) = 0$

$\Rightarrow \quad 16k^2 - 4k^2 + 4k - 8 = 0$

$\Rightarrow \quad 12k^2 + 4k - 8 = 0$ *i.e.* $3k^2 + k - 2 = 0$

$\Rightarrow \quad 3k^2 + 3k - 2k - 2 = 0$ *i.e.* $3k(k + 1) - 2(k + 1) = 0$

$\Rightarrow \quad (k + 1)(3k - 2) = 0$ *i.e.* $k + 1 = 0$ or $3k - 2 = 0$

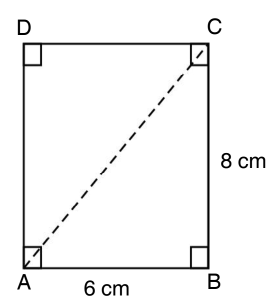
$\Rightarrow \quad \boldsymbol{k = -1}$ or $\boldsymbol{k = \frac{2}{3}}$ **Ans.**

(b) On a map drawn to a scale of 1 : 50,000, a rectangular plot of land ABCD has the following dimensions, AB = 6 cm; BC = 8 cm and all angles are right angles. Find :

(i) the actual length of the diagonal distance AC of the plot in km.

(ii) the actual area of the plot in sq km. [3]

Solution :



Given scale factor, $k = \frac{1}{50,000}$

(i) Δ ABC is right angled at vertex B.

$\Rightarrow \quad AC^2 = AB^2 + BC^2$

$= 6^2 + 8^2$

$= 36 + 64 = 100$

$\therefore \quad AC = 10$ cm [On the map]

$\because$ Length of AC on map = $k \times$ actual length of AC

$\Rightarrow \quad 10 \text{ cm} = \frac{1}{50,000} \times$ actual length of AC

$\Rightarrow$ **Actual length of AC** $= 10 \times 50{,}000$ cm

$$= \frac{10 \times 50{,}000}{100 \times 1{,}000} \text{ km} = \mathbf{5 \text{ km}} \qquad \textbf{Ans.}$$

(ii) $\because$ Area of rectangular plot ABCD on map = 6 cm × 8 cm = 48 cm^2

And, area on map $= k^2 \times$ actual area

$$\Rightarrow \quad 48 \text{ cm}^2 = \left(\frac{1}{50{,}000}\right)^2 \times \text{Actual area of plot}$$

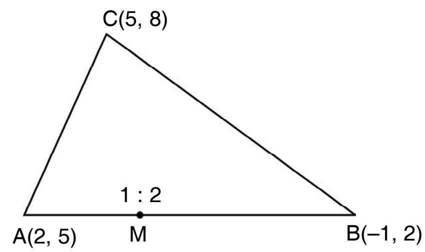
$$\Rightarrow \quad 48 \times 50{,}000 \times 50{,}000 \text{ cm} = \text{Actual area of plot}$$

$$\Rightarrow \quad \textbf{Actual area of plot} = \frac{48 \times 50{,}000 \times 50{,}000}{(100 \times 1{,}000)^2} \text{ km}^2$$

$$= \mathbf{12 \text{ sq. km}} \qquad \textbf{Ans.}$$

(c) A(2, 5), B(–1, 2) and C(5, 8) are the vertices of a triangle ABC, 'M' is a point on AB such that AM : MB = 1 : 2. Find the co-ordinates of 'M'. Hence find the equation of the line passing through the points C and M. [4]

Solution :



$$\because \frac{AM}{MB} = \frac{1}{2} \Rightarrow \frac{m_1}{m_2} = \frac{1}{2}$$

$$\therefore \quad \mathbf{M} = \left(\frac{1 \times -1 + 2 \times 2}{1+2}, \frac{1 \times 2 + 2 \times 5}{1+2}\right)$$

$$= \left(\frac{3}{3}, \frac{12}{3}\right) = \mathbf{(1, 4)} \qquad \textbf{Ans.}$$

$\because$ C = (5, 8) and M = (1, 4)

Slope of line through C and M

$$= \frac{4-8}{1-5} = \frac{-4}{-4} = 1 \quad i.e. \quad m = 1$$

Equation of CM is :

$$y - y_1 = m(x - x_1)$$

$\Rightarrow \quad y - 4 = 1(x - 1)$ [Taking (x_1, y_1) = M(1, 4) and slope (m) of CM = 1]

$\Rightarrow$ $\boldsymbol{x - y + 3 = 0}$ **Ans.**

Question **8 :**

(a) ₹ 7500 were divided equally among a certain number of children. Had there been 20 less children, each would have received ₹ 100 more. Find the original number of children.

Solution :

Let the original number of children = x

$\therefore$ Each of children gets $\frac{₹\ 7500}{x}$

If number of children = $x - 20$

$$\text{Each child gets} = \frac{₹\,7500}{x-20}$$

$$\text{Given : } \frac{₹\,7500}{x-20} - \frac{₹\,7500}{x} = ₹\,100$$

$$\Rightarrow \quad \frac{7500x - 7500(x-20)}{(x-20)\times x} = 100$$

$\Rightarrow \quad 100(x^2 - 20x) = 7500x - 7500x + 1,50,000$

$\Rightarrow \quad 100(x^2 - 20x) = 150000$

$\Rightarrow \quad x^2 - 20x = 1500$ *i.e.* $x^2 - 20x - 1500 = 0$

$\Rightarrow \quad x^2 - 50x + 30x - 1500 = 0$

$\Rightarrow \quad x(x - 50) + 30(x - 50) = 0$

$\Rightarrow \quad (x - 50)(x + 30) = 0$ *i.e.* $x = 50$ or $x = -30$

$\Rightarrow \quad x = 50$

i.e. **the original number of children = 50** **Ans.**

Alternative method :

Let orginal number of children = x

$$\Rightarrow \quad \text{Each of the children gets} = \frac{₹\,7500}{x}$$

When the number of children is 20 less

Number of children = $x - 20$

$$\text{Now each of children gets} = ₹\left(\frac{7500}{x} + 100\right)$$

$$\therefore \quad (x-20)\left(\frac{7500}{x} + 100\right) = 7500$$

$$\Rightarrow \quad (x-20) \times \frac{7500 + 100x}{x} = 7500$$

$\Rightarrow \quad 7500x + 100x^2 - 150000 - 2000x = 7500x$

$\Rightarrow \quad 100x^2 - 2000x - 150000 = 0$

$\Rightarrow \quad x^2 - 20x - 1500 = 0$

On solving we get $x = 50$ or $x = -30$

$\Rightarrow$ $x = 50$ *i.e.* **the original number of children = 50** **Ans.**

(b) If the mean of the following distribution is 24, find the value of '*a*'. **[3]**

Marks	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**
No. of students	**7**	***a***	**8**	**10**	**5**

Solution :

Marks	Number of students (f)	Class mark (x)	$f \times x$
0-10	7	5	35
10-20	a	15	$15a$
20-30	8	25	200
30-40	10	35	350
40-50	5	45	225
	$\Sigma f = 30 + a$		$\Sigma fx = 810 + 15a$

$\because$ $\quad$ Mean $= \dfrac{\sum fx}{\sum f}$

$\Rightarrow$ $\quad 24 = \dfrac{810+15a}{30+a}$

$\Rightarrow$ $\quad 720 + 24a = 810 + 15a$

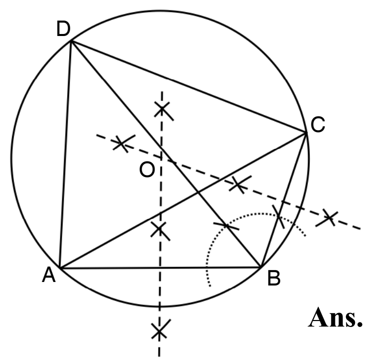
$\Rightarrow$ $\quad 9a = 90$ and $\boldsymbol{a = 10}$ **Ans.**

(c) Using ruler and compass only, construct a ΔABC such that BC = 5 cm, AB = 6·5 cm and ∠ABC = 120°.

(i) Construct a circum-circle of ΔABC.

(ii) Construct a cyclic quadrilateral ABCD, such that D is equidistant from AB and BC. **[4]**

Solution :



(i) With given dimensions construct the ΔABC. Draw perpendicular bisectors of AB and BC which meet each other at point O. With O as centre and OA or OB or OC as radius, draw a circle which passes through vertices A, B and C.

$\therefore$ **The circle so drawn is the circum-circle of triangle ABC.** **Ans.**

(ii) D is equidistant from AB and BC

$\Rightarrow$ D is on bisector of ∠ABC

$\therefore$ Draw bisector of ∠ABC which meets the circum-circle drawn at point D. Join DA and DC.

$\therefore$ **ABCD is the required cyclic quadrilateral.** **Ans.**

***Question* 9 :**

(a) Priyanka has recurring deposit account of ₹ 1000 per month at 10% per annum. If she gets ₹ 5550 as interest at the time of maturity, find the total time for which the account was held.

Solution :

Given : Amount deposited per month, P = ₹ 1000, rate of interest, r = 10% and interest earned, I = ₹ 5550

To find, time, n.

$\because$ $$I = P \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}$$

$\Rightarrow$ $$5550 = ₹\, 1000 \times \frac{n(n+1)}{24} \times \frac{10}{100}$$

i.e. $1332 = n^2 + n$

$\Rightarrow$ $n^2 + n - 1332 = 0$

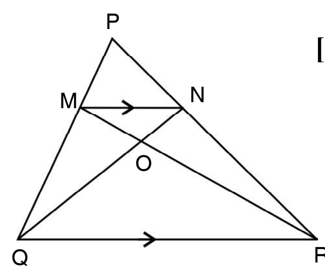
i.e. $x^2 + 37n - 36n - 1332 = 0$

$\Rightarrow$ $n(n + 37) - 36(n + 37) = 0$

i.e. $n = -37$ or $n = 36$

$\therefore$ **Required time**, $n = 36$ months = **3 years** **Ans.**

(b) In ΔPQR, MN is parallel to QR and $\frac{PM}{MQ} = \frac{2}{3}$ **[3]**

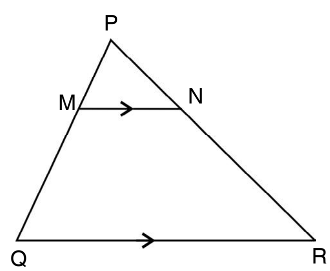


(i) Find $\frac{MN}{QR}$.

(ii) Prove that ΔOMN and ΔORQ are similar.

(iii) Find, Area of ΔOMN : Area of ΔORQ.

Solution :



(i) MN // QR

$\therefore$ In ΔPMN and ΔPQR

$\Rightarrow$ $\angle PMN = \angle PQR$ (Corresponding angles)

$\angle PNM = \angle PRQ$ (" " " ")

and $\angle P$ is common

$\therefore$ $\Delta PMN \sim \Delta PQR$ (by A.A.A.)

$$\Rightarrow \frac{\mathbf{MN}}{\mathbf{QR}} = \frac{PM}{PQ}$$

$$= \frac{PM}{PM + MQ} = \frac{2}{2+3} = \mathbf{\frac{2}{5}}$$ **Ans.** $\left[\because \frac{PM}{MQ} = \frac{2}{3}\right]$

(ii) In triangles OMN and ORQ :

$\angle MON = \angle QOR$ (Vertically opposite angles)

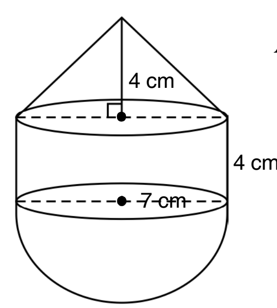
and $\angle MNO = \angle RQO$ (Alternate angles)

$\Rightarrow$ **ΔOMN and ΔORQ are similar.** (by A.A.) **Ans.**

(iii) Since, the areas of two similar triangles are in proportion to the squares of their corresponding sides

$$\therefore \frac{\mathbf{Ar.(\Delta OMN)}}{\mathbf{Ar.(\Delta ORQ)}} = \frac{MN^2}{QR^2} = \frac{2^2}{5^2} = \mathbf{\frac{4}{25}}$$ **Ans.**

(c) The adjoining figure represents a solid consisting of a right circular cylinder with a hemisphere at one end and a cone at the other. Their common radius is 7 cm. The height of the cylinder and cone are each of 4 cm. Find the volume of the solid. **[4]**



Solution :

The volume of the given solid

= Volume of cone + volume of cylinder + volume of hemi-sphere

$$= \frac{1}{3}\pi r^2 h + \pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 4 + \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 4 + \frac{2}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^3$$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \left[\frac{4}{3} + 4 + \frac{2}{3} \times \frac{7}{2}\right] \text{ cm}^3$$

$$= \frac{77}{2} \times \frac{23}{3} \text{ cm}^3 = \mathbf{295{\cdot}17 \text{ cm}^3}$$ **Ans.**

***Question* 10 :**

(a) Use Remainder theorem to factorize the following polynomial :
$2x^3 + 3x^2 - 9x - 10$. **[3]**

Solution :

Substituting $x = -1$, we get :

Remainder $= 2 \times (-1)^3 + 3 \times (-1)^2 - 9 \times -1 - 10$

$= -2 + 3 + 9 - 10 = 0$

$\Rightarrow$ $(x + 1)$ is a factor of given polynomial

$2x^3 + 3x^2 - 9x - 10$

$= (x + 1)(2x^2 + x - 10)$

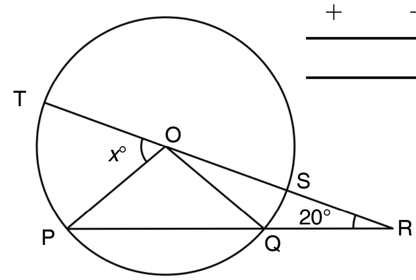
$= (x + 1)(2x^2 + 5x - 4x - 10)$

$= (x - 1)[x(2x + 5) - 2(2x + 5)]$

$= \mathbf{(x + 1)(2x + 5)(x - 2)}$ **Ans.**

$$\begin{array}{r|l} & 2x^2 + x - 10 \\ \hline x + 1 & 2x^3 + 3x^2 - 9x - 10 \\ & 2x^3 + 2x^2 \\ & - \quad - \\ \hline & x^2 - 9x - 10 \\ & x^2 + x \\ & - \quad - \\ \hline & -10x - 10 \\ & -10x - 10 \\ & + \quad + \\ \hline & \times \\ \hline \end{array}$$

(b) In the given figure, 'O' is the centre of the circle. If QR = OP and ∠ORP = 20°, find the value of '*x*' giving reasons. **[4]**



Solution :

Given QR = OP

and OP = OQ

= Radii of the given circle

∴ QR = OQ

$\Rightarrow$ ∠QOR = ∠ORQ = 20°

In ΔOQR,

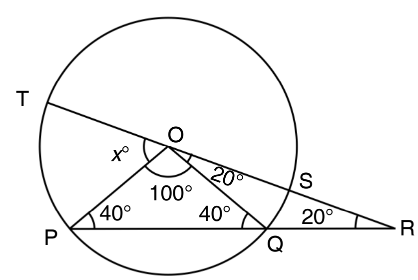
ext. angle OQP = 20° + 20° = 40°

OP = OQ

$\Rightarrow$ ∠OPQ = ∠OQP = 40°

And, ∠OPQ + ∠OQP + ∠POQ = 180°

$\Rightarrow$ 40° + 40° + ∠POQ = 180° *i.e.* ∠POQ = 100°

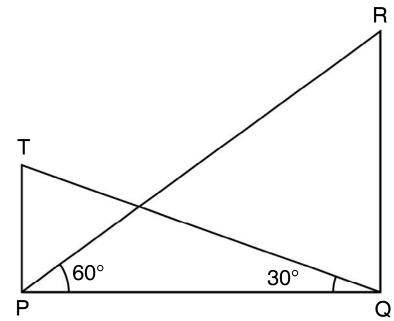


TOS is a straight line

$\Rightarrow \quad x + 100° + 20° = 180° \quad i.e. \quad x = \mathbf{60°}$ **Ans.**

> Also, since exterior angle of a triangle is equal to sum of its interior opposite angles.
> $\therefore$ In ΔOPR, $x° = 40° + 20° = \mathbf{60°}$.

(c) The angle of elevation from a point P of the top of a tower QR, 50 m high is 60° and that of the tower PT from a pont Q is 30°. Find the height of the tower PT, correct to the nearest metre. [4]



Solution :

In ΔPQR, $\frac{PQ}{QR} = \cot 60°$

i.e. $\frac{PQ}{50\,m} = \frac{1}{\sqrt{3}} \Rightarrow PQ = \frac{50}{\sqrt{3}}$ m

In ΔTPQ, $\frac{PT}{PQ} = \tan 30°$

i.e. **Height of the tower PT** $= \frac{1}{\sqrt{3}} \times PQ$

$= \frac{1}{\sqrt{3}} \times \frac{50}{\sqrt{3}}$ m

$= \frac{50}{3}$ m $= 16{\cdot}67$ m $=$ **17 m** **Ans.**

***Question* 11 :**

(a) The 4th term of an A.P. is 22 and 15th term is 66. Find the first term and the common difference. Hence, find the sum of the series to 8 terms. [4]

Solution :

Let the first term = a and common difference = d

Given : 4th term = 22 $\Rightarrow$ $a + 3d = 22$ I

and, 15th term = 66 $\Rightarrow$ $a + 14d = 66$ II

On solving the two equations, we get : $a = 10$ and $d = 4$

Sum of 8 terms $= \frac{8}{2}[2 \times 10 + (8 - 1) \times 4]$

$= 4[20 + 28] = \mathbf{192}$ **Ans.**

(b) Use graph paper for this question.

A survey regarding height (in cm) of 60 boys belonging to class 10 of a school was conducted. The following data was recorded :

Height in cm	135-140	140-145	145-150	150-155	155-160	160-165	165-170
No. of boys	4	8	20	14	7	6	1

Taking 2 cm = height of 10 cm along one axis and 2 cm = 10 boys along the other axis draw an ogive of the above distribution. Use the graph to estimate the following :

(i) the median. (ii) lower quartile.

(iii) if above 158 cm is considered as the tall boys of the class, find the number of boys in the class who are tall. [6]

Solution :

Height in cm (C.I.)	135-140	140-145	145-150	150-155	155-160	160-165	165-170
No. of boys (*f*)	4	8	20	14	7	6	1
c.f.	4	12	32	46	53	59	60

According to the given scales, plot the points (140, 4), (145, 12), (150, 32), (155, 46), (160, 53), (165, 59) and (170, 60) on a graph paper.

Join all the points, plotted on the graph, with a free-hand curve starting from the lower limit (135) of the first class (135-140) and terminating at the upper limit (170) of the last class (165-170). The resulting graph will be an ogive as shown below :

(i) **The median** = Height of $\left(\frac{60}{2}\right)^{th}$ 1 boy

= Height of 30th boy = **149·5 cm.** **Ans.**

(ii) **Lower quartile** = Height of $\left(\frac{60}{4}\right)^{th}$ boy

= Height of 15th boy = **146 cm** **Ans.**

(iii) Since, number of boys with heights upto 158 cm = 50

∴ **The number of tall boys** = Number of boys with heights more than 158 cm

= 60 − 50 = **10** **Ans.**